We examine quantity-setting behavior in the presence of anti-dumping law in an infinite horizon international duopoly model. Firms’ quantity setting for the current period affects the expected anti-dumping duty levied on imports in the next period. Therefore, firms decide their individual output levels taking into account their impact on strategic interactions from the next period onwards. By considering hypothetically the impact of firms’ current output decisions only on their strategic positions in the next period, and ignoring effectively subsequent periods, a useful understanding about the relationship between two period and infinite horizon formulations can be gained.

**JEL classification numbers** C73, F13, L13

**Key Words** Infinite Horizon Dynamic Duopoly, Dumping, Anti-Dumping Laws, Markov Perfect Equilibrium, Strategic Effect
1. Introduction

Based on Article VI of GATT, "dumping", where the products of one country are introduced into the market of another country at less than their “normal” value, is to be condemned if it causes or threatens material injury (damage) to an established industry. A product is regarded as having been dumped into the market of an importing country, when the price of the product exported from one country to another is less than the price of the same product intended for consumption in the exporting country. Accordingly, dumping “margin” is defined as the difference between a “normal” or “fair” value for the product and the price charged for it in the market of an importing country, and so there is a positive dumping margin whenever the export price is below the “normal” value.

According to casual observation in the U.S. market, numerous goods, companies, and countries are subject repeatedly to the allegation of dumping. Welded carbon steel from Taiwan, butt-weld pipe fittings from Japan and orange juice from Brazil are examples.

Recently, the Japanese steel industry has been heavily attacked by the protectionist U.S. steel industry and government (Department of Commerce) through the anti-dumping provisions. There seems to be no end to anti-dumping suits against Japanese steel products. The Chinese government also has recently started an anti-dumping investigation into the Japanese chemical products, while Chinese color TV products are about to subject to dumping litigation in the U.S. market.

Nevertheless, neither exclusion of foreign goods nor price equalization is necessarily an observed outcome of dumping suits in the United States. The fact that lawsuits are repeatedly brought suggests prices do not equalize between countries. Further, the anti-dumping laws seem not to have excluded firms from the market. Indeed, such a result is at least not the stated intent of the anti-dumping laws. Foreign (for example, Japanese, Taiwanese, and Chinese) steel has been sold in the United States, as are Toyota mini-vans.

One explanation for these phenomena could be that the optimal behavior of firms, confronted with the United States’ anti-dumping legal structure, is to continue a dynamic strategy that leaves the firms susceptible to dumping charges.

The legal structure of dumping statutes is well suited to a stochastic game scenario. A firm decides output in a period knowing the dumping margin in the previous period, but must decide output for this period without knowing the actual duty that might be imposed because of delays in litigation. Since its actions today influence certain aspects of the dumping margin in the next period, the firm cannot behave independently of the next period.

We present a model that captures the dual role of the dumping margin and the stochastic nature of anti-dumping law enforcement. The effect of anti-dumping laws on foreign and domestic firms' dynamic behavior will be examined. Then, we explicitly build into the model both the initial direct
impact of the dumping behavior and the probability of a positive initial finding (authorization of the fact) that imposes the positive anti-dumping duties at the end of the next period.

An anti-dumping assessment (charge) is an instrument of the home firm, which is often realized through its political pressure on the government. But, the calculation of the dumping margin is such that the result is not at all certain. Enforcement of current anti-dumping law therefore becomes dependent on a bureaucratic and political process, which leads to a high degree of uncertainty regarding outcome. The effectiveness of the World Trade Organization (WTO) as an adjustment (settlement) mechanism for international trade disputes is still unclear.

We will consider the outcome of this political/negotiation process as a probability function\(^1\) reflecting the failure rate associated with anti-dumping filings, data gatherings, and political negotiations among the several interested parties. These elements will also lead to delay in litigation. Such an institutional structure surrounding the determination and administration of AD duties, that is, anti-dumping protection policy with a lagged administrative review process will make the analysis of this kind of trade policy interesting and complex.

We aim at analyzing a model over an infinite time horizon, and our (-main-) solution concept is the pure strategy Markov Perfect Equilibrium (MPE). We want to use a model and solution concept where profits in the current period are not independent of actions in the previous period. The model we developed and the solution concept used are specifically designed to incorporate this lack of time independence.

In order to justify the use of the MPE, we shall refer to the existing literature. First, Fudenberg and Tirole (1991) (pp.501) state that "When studying more complex environments, economists often focus attention on equilibria in a smaller class of 'Markov' or 'state-space' strategies in which the past influences current play only through its effect on a state variable that summarizes the direct effect of the past on the current environment." This states concisely our motivation in choosing the MPE solution concept. Second, in their "Theory of Dynamic Oligopoly" papers, Maskin and Tirole (e.g. their1987 paper) justified the use of the MPE on several grounds. One of them is \textit{simplicity}. The MPE removes many possible strategies: They further justify the use of the MPE on the grounds that Markov strategies seem at times to accord better with the conventional conception of a reaction in the informal I.O literature than do, say, the reactions emphasized in the repeated game tradition. In other words, we can say that the Markov Perfect Equilibrium (MPE) does rule out some potentially important interactions between past behavior and current actions, but it may represent a reasonable compromise between the need for simplification and the need to allow for dynamic interaction.

With regard to the legal structure of dumping statutes, explained above, because of delays in litigation, a firm decides output for a given period, knowing the dumping margin in the previous

\(^1\) We incorporate into the model this bureaucratic process of the enforcement of anti-dumping laws, as a reduced form.
period, but without knowing the actual duty in this period, in other words, decides output in a period, given an expected anti-dumping duty in that period. This structure gives a justification for making use of the Markov perfect (closed-loop) equilibrium in the specific context of the anti-dumping laws.

Under such motivation, we analyze an infinite-horizon international-duopoly model that captures the dual role of the dumping margin and the stochastic nature of anti-dumping law enforcement. We solve for a Markov perfect (closed-loop) Equilibrium, where the state variable is an expected anti-dumping duty in effect during a given period. A strategic effect by changing the opposite firm’s future output choices through the impact of the firm’s current output choices on future anti-dumping duties shall also be considered.¹

We examine quantity-setting behavior in the presence of anti-dumping law in an infinite horizon international duopoly model. Firms’ quantity setting for the current period affects the expected anti-dumping duty levied on imports in the next period. Therefore, firms decide their individual output levels taking into account their impact on strategic interactions from the next period onwards. By hypothetically considering the impact of firms’ output decisions in the current period on their strategic positions only in the next period, and effectively ignoring those in subsequent periods, a useful understanding about the relationship between two period and infinite horizon formulations can be gained.

Related Literature

The literature on dumping and anti-dumping laws is quite extensive. Economically, dumping can be based on a price comparison or based on a price below cost (average cost), and so dumping margin is either price or cost based. One possible source of dumping margin is price discrimination. If an exporting country protects its own firm from competition while that firm can compete as a duopolist in an importing country and if demand in the two countries is roughly similar, then we would expect a higher price in the monopolized exporting country than in the duopolistic importing country. This is price discrimination and can cause a price difference that could be regarded as a dumping margin. Our paper uses this setting as a static framework, and extends it to a dynamic one.

There is an alternative stream of literature that focuses on dumping occurring when a foreign firm sells goods in the home market for less than cost. For instance, Staiger and Wolak (1992) uses capacity constraint to explain the cost based dumping margin. In their paper, the anti-dumping law diminishes dumping activity without a suit being filed, and dumping is a result of stochastic demand with capacity constraints.

¹ Prusa (1994) uses a two-period oligopoly model to address how firms would react in the first period to the possibility of anti-dumping duties in the second period. Our paper has a theoretical value added in the sense of using a dynamic oligopoly setting and the solution concept of Markov Perfect Equilibrium (MPE), and also analyzing in a more detailed way the strategic effects of the anti-dumping law.
Next, we refer to recent related literature. Blonigen and Park’s (2004) study resembles our own. They examine the dynamic pricing problem that a foreign firm faces in the presence of AD investigations and duties in the export market. They also highlight the anti-dumping policy with a lagged administrative review process, and emphasize that the enforcement of anti-dumping policy is uncertain. A difference between their paper and ours is that theirs analyzes the dynamic optimization problem faced by the foreign firm in the above environment, not a dynamic “games” framework, while our paper analyzes an infinite horizon dynamic duopoly environment, and strategic incentives used both by the foreign firm and the home firm. Another related paper is Harrington (2002). He introduces both the probability of detection and the antitrust penalties in the form of specified exogenous (reduced form) functions into a repeated game model of collusion, and investigates the effect of “antitrust laws” on the firms’ dynamic pricing behaviors, in other words, the internal stability of the cartel.

2. The Model

2.1 Setting

Consider two countries (home and foreign), with one firm in each country. The firms choose output $X_i$ ($i = F$ for foreign output sold in the foreign market, $f$ for foreign output sold at home, $H$ for home output sold at home) to maximize profit in infinite discrete time. They produce a single homogeneous product at identical, constant marginal costs. For simplicity, we assume that the marginal cost in both countries is 0. The home and foreign markets are segmented, with the home firm excluded from the foreign market. Demand is identical in both markets. This gives a simple dumping model based on price discrimination.

The foreign firm's output is subject to an anti-dumping duty in the following manner. Any positive difference between the price in the foreign market and the price at home is considered a dumping margin. There is a probability distribution function $f(P_F - P_H)$ of being successfully charged with dumping, where $P_F(X_F)$ is inverse demand in the foreign market (e.g. Japan), and $P_H(X_f, X_H) = P_H(X_f + X_H)$ is inverse demand in the home market (e.g., the United States). $f(\cdot)$ is non-decreasing in the dumping margin: $P_F - P_H$, that is, $f'(\cdot) \geq 0$. The formulation of the above probability function implies that even with knowledge of the legal constructs of

\footnote{For example, we can imagine that the home firm is viewed as the American firm (A firm), and the foreign firm is viewed as the Japanese firm (J firm). We assume that the American firm is excluded from the Japanese market through various forms of entry barriers.}
anti-dumping laws, whether or not the dumping behavior is found is the result of a process of data gathering and averaging, which creates a good deal of uncertainty.

Next, we denote the size of the anti-dumping duty as \( g(P_f - P_h) \). \( g(\cdot) \) is increasing in the dumping margin, and is increasing at a constant or increasing rate, that is, \( g'(\cdot) \geq 0, g''(\cdot) \geq 0 \).

This type of uncertainty addresses both the data gathering problems of the dumping margin and the consideration of the inconsistency of anti-dumping law enforcement. This function represents the relationship between the magnitude of the dumping margin and the magnitude of the imposed anti-dumping duties.

Hence, \( h_t = f \left( P_{F,t-1} - P_{H,t-1} \right) g \left( P_{F,t-1} - P_{H,t-1} \right) \) is the expected anti-dumping duty in period \( t \), given a dumping margin in period \( t-1, P_{F,t-1} - P_{H,t-1} \). This means that the dynamics are modeled by \( h_t = f \left( P_{F,t-1} - P_{H,t-1} \right) g \left( P_{F,t-1} - P_{H,t-1} \right) \) \( t = 1, 2, \ldots \) and we assume that this structure itself is time invariant (stationary). We also assume that \( h_t(0) = f_t(0) = g_t(0) = 0 \) and \( h'(0) \geq 0 \), and that \( h_t \) is non-negative and non-decreasing in the dumping margin. \( h_t \) is applied to a foreign firm’s sales at home during period \( t \). Firms arrive at period \( t \) knowing the dumping margin in \( t-1 \) and therefore \( h_t \). Any anti-dumping duty based on the margin is applied to goods sold in period \( t \). At the end of the market in period \( t \), firms incur the actual duty. A firm subject to the duty will have an incentive to reduce output in order to avoid some portion of the duty (See, Reizes (1993)). By doing so, they will change the dumping margin. Therefore, firms cannot adjust output in order to avoid a duty without affecting the expected anti-dumping duty in the next period. This structure captures the stochastic nature of enforcement of anti-dumping law, due to the lagged administrative review process.

### 2.2 Game Representation

This situation has the structure of stochastic dynamic games. Let us review it. Players are the foreign firm and the home firm. Their actions for each period \( t = 1, 2, \ldots \) are \( \left(X_{F,t}, X_{H,t}\right) \) for the foreign firm and \( X_{H,t} \) for the home firm. The state variable at each period is an expected anti-dumping duty, given in that period. If the state variable in a given period \( t \) has value \( h_t \), and

\[ h_t(0) = f_t(0) = g_t(0) = 0 \text{ and } h'(0) \geq 0, \]

As we explained in detail, \( h(P_f - P_h) = g(P_f - P_h)g(P_f - P_h) \) represents the expected anti-dumping duty on a forward-looking basis. Firms arrive at the market, knowing \( h_t \). But, we assumed
the vector of actions chosen by players is \( X_t = (X_{f,t}, X_{h,t}, X_{H,t}, X_{F,t}) \), then the actual anti-dumping duty imposed at the end of this period is a random variable, under uncertainty of anti-dumping law enforcement, and the state variable in the next period is

\[
h_{t+1} = f(P_{f,t} - P_{h,t})g\left(P_{f,t} - P_{h,t}\right)
\]

as the expected anti-dumping duty in the next period, when the dumping margin in period \( t \) is \( P_{f,t} - P_{h,t} \).

The payoff to the foreign firm at stage game in period \( t \) when the state is \( h_t \) is

\[
\Pi_{F,t}(h_t, (X_{F,t}, X_{f,t}), X_{H,t}) = P_{f,t}(X_{F,t})X_{f,t} + P_{H,t}(Q_{H,t})X_{f,t} - h_tX_{f,t}
\]

and similarly, the payoff to the home firm at stage game in period \( t \) is

\[
\Pi_{H,t}(h_t, (X_{F,t}, X_{f,t}), X_{H,t}) = P_{H,t}(Q_{H,t})X_{H,t} = P_{H,t}(X_{f,t} + X_{H,t})X_{H,t}
\]

where \( Q_{H,t} = X_{f,t} + X_{H,t} \) is total output sold at home in period \( t \).

In the infinite horizon game, since both the foreign firm and the home firm maximize the discounted expected profits, the payoff functions are respectively,

\[
\sum_{t=1}^{\infty} \delta^{t-1} \Pi_{F,t}(h_t, (X_{F,t}, X_{f,t}), X_{H,t}) \quad \text{and} \quad \sum_{t=1}^{\infty} \delta^{t-1} \Pi_{H,t}(h_t, (X_{F,t}, X_{f,t}), X_{H,t})
\]

where \( \delta \in [0,1) \) is a common discount factor.

### 2.3 Equilibrium Concepts

The equilibrium concept that we mainly adopt is a pure strategy Markov Perfect Equilibrium. The strategy for players \( H \) (home firm), \( F \) (foreign firm) is, respectively,

\[
X_{H,t}(h_t), (X_{F,t}(h_t), X_{f,t}(h_t)), t = 1, 2, \cdots,
\]

where \( X_{i,t}(h_t) \) is the Markov strategy of player \( i = H, F \), in that strategies depend only on a state variable \( h_t \).

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that only after the market in that period did firms realize the actual duty. Under this information structure, \( h_t \) is the state variable at the beginning of period \( t \).
Definition: A pair of strategies \( \{ X_{H,t}^*, (X_{F,t}^* (h_t), X_{f,t}^* (h_t)) \} \), \( t = 1, 2, \cdots \) is called a Markov Perfect Nash Equilibrium (MPE) of the dynamic game if for every feasible state \( h_t \) at time period \( t = 1, 2, \cdots \), we have for every feasible pair \( (X_{H,t}^* (h_t), (X_{F,t}^* (h_t), X_{f,t}^* (h_t))) \), \( t = 1, 2, \cdots \)

\[
\sum_{k=1}^{\infty} \delta^{k-1} \Pi_{H,k} (X_{H,k}^* (h_k), X_{F,k}^* (h_k), X_{f,k}^* (h_k)) \geq \sum_{k=1}^{\infty} \delta^{k-1} \Pi_{H,k} (X_{H,k}^* (h_k), (X_{F,k}^* (h_k), X_{f,k}^* (h_k)))
\]

\[
\sum_{k=1}^{\infty} \delta^{k-1} \Pi_{F,k} (X_{F,k}^* (h_k), X_{F,k}^* (h_k), X_{f,k}^* (h_k)) \geq \sum_{k=1}^{\infty} \delta^{k-1} \Pi_{F,k} ((X_{F,k}^* (h_k), X_{f,k}^* (h_k)), X_{H,k}^* (h_k))
\]

In summary, \( \{ X_{H,t}^* (h_t), (X_{F,t}^* (h_t), X_{f,t}^* (h_t)) \} \), \( t = 1, 2, \cdots \) is said to be a Markov Perfect Equilibrium (MPE) if and only if for every player \( i = H, F \), at every state \( h_t \) at time period \( t = 1, 2, \cdots \), the player would find no incentive to deviate from the equilibrium strategies, as far as the other player follows them. In this equilibrium concept, the play to follow after every state \( h_t \) prescribes a Nash equilibrium for the game that starts at \( h_t \), which is commonly referred to as a subgame. In that sense, since the play off the equilibrium path is credible, this solution concept is time consistent. Hence, we can say that a Markov Perfect Nash Equilibrium (MPE) is a subgame perfect Nash equilibrium, where strategies depend only on specified state variables.

We also examine the “Open-Loop” equilibrium of the game, in order to compare the equilibrium incentives. It is a “Nash Equilibrium” of the dynamic game, and each player \( i = H, F \) commits himself to a future path once at the beginning of the game, and no player has an incentive to deviate by playing another feasible path from the initial state \( h_t \), as far as the other player follows. However, the play prearranged after some state other than initial state \( h_t \) may not constitute itself a Nash equilibrium for the subgame that starts at such state. In other words, this solution concept is not time consistent (subgame perfect). Each player ignores the evolution of the state variable in the game, and does not optimally respond to each state \( h_t \) caused by the political uncertainty (i.e., to the expected anti-dumping duty \( h_t \) in each period). It seems to be rather irrational. Thus, in order to avoid non-credible equilibria that may not prescribe equilibrium play after a subsequent state \( h_t \), we use Markov Perfect Equilibrium (MPE) as a main equilibrium concept.

Hereafter, we analyze the different formulations and equilibria between two-period and infinite horizon cases. For a two period model, the above definition still applies by simply defining the payoff function to be the discounted present value of the expected payoffs over two periods.
2.4 Two Period Formulation

We will start by examining the two period version of this model. We define the home firm's problem (in period 1) as:

\[ V_{H,1} \left( h \left( P_{F,0} - P_{H,0} \right) \right) = \max_{X_{H,1}} P_{H,1} X_{H,1} + \delta V_{H,2} \left( h \left( P_{F,1} - P_{H,1} \right) \right) \]

where \( V_{H,2} \left( h \left( P_{F,1} - P_{H,1} \right) \right) = \max_{X_{H,2}} P_{H,2} X_{H,2} \)

Let \( h_2 = h \left( P_{F,1} - P_{H,1} \right) \) denote the level of the state variable in period \( t = 2 \). Differentiating \( V_{H,1} \) with respect to \( X_{H,1} \) yields:

\[ \frac{\partial V_{H,1}}{\partial X_{H,1}} = P_{H,1} X_{H,1} + P_{H,1} + \delta \left( \frac{\partial V_{H,2}}{\partial X_{H,1}} \right) = 0 \]

where \( \frac{\partial V_{H,2}}{\partial X_{H,1}} = P_{H,1} \left( -h'_2 \right) \frac{dX_{f,2}}{dh_2} P_{H,2} X_{H,2} \)

The interpretation is as follows. An increase in the home firm's current output \( X_{H,1} \) lowers the price in the home market, which increases the dumping margin \( P_{F,1} - P_{H,1} \). It increases both the probability that anti-dumping duties will be in effect in the next period and the magnitude of those duties if enforcement is successful. Through its impact on future expected anti-dumping duties, the home firm’s current output choice \( X_{H,1} \) can influence the foreign firm’s future output choice \( X_{f,2} \).

That is, it increases the expected anti-dumping duty (state \( h_2 \)) in the next period, and induces in equilibrium less aggressive (passive) behavior by the foreign firm, which will increase the profit of the home firm in the home market. This is a positive strategic effect.

Similarly, the foreign firm's problem can be written, for some arbitrary period, \( t = 1 \), as:

\[ V_{F,1} \left( h \left( P_{F,0} - P_{H,0} \right) \right) = \max_{X_{F,1},X_{f,1}} P_{F,1} X_{F,1} + P_{H,1} X_{f,1} - h \left( P_{F,0} - P_{H,0} \right) X_{f,1} + \delta V_{F,2} \left( h \left( P_{F,1} - P_{H,1} \right) \right) \]

where \( V_{F,2} \left( h \left( P_{F,1} - P_{H,1} \right) \right) = \max_{X_{F,2},X_{f,2}} P_{F,2} X_{F,2} + P_{H,2} X_{f,2} - h \left( P_{F,1} - P_{H,1} \right) X_{f,2} \)

Let \( h_1 = h \left( P_{F,1} - P_{H,1} \right) \) denote the level of the state variable in period \( t = 1, 2 \). Differentiating with respect to \( X_{F,1} \) and \( X_{f,1} \) yields:
\[ \frac{\partial V_{F,1}}{\partial X_{F,1}} = P'_{F,1}X_{F,1} + P_{F,1} + \delta \left( \frac{\partial V_{F,2}}{\partial X_{F,1}} \right) = 0 \]  
(2)

\[ \frac{\partial V_{H,1}}{\partial X_{F,1}} = P'_{H,1}X_{F,1} + P_{H,1} - h_1 + \delta \left( \frac{\partial V_{F,2}}{\partial X_{F,1}} \right) = 0 \]  
(3)

where

\[ \frac{\partial V_{F,2}}{\partial X_{F,1}} = -P'_{F,1}h_2'X_{f,2} + P'_{F,1}h_2' \frac{dX_{H,2}}{dh_2} P'_{H,2}X_{f,2} \]  
(4)

\[ \frac{\partial V_{H,2}}{\partial X_{F,1}} = P'_{H,1}h_2'X_{f,2} + P'_{H,1}(-h_1') \frac{dX_{H,2}}{dh_2} P'_{H,2}X_{f,2} \]  
(5)

The envelope theorem was made use of in the derivation of (4) and (5). The interpretation of (4) is two-fold. First, an increase in the foreign firm’s current output \( X_{F,1} \) lowers the price in the foreign market, which decreases the dumping margin \( P_{F,1} - P_{H,1} \). It decreases both the probability that anti-dumping duties will be in effect in the next period and the magnitude of those duties if enforcement is successful. This brings about a positive direct effect through reducing the impact on future expected anti-dumping duty (state \( h_2 \)), corresponding to the first term of (4). Second, an increase in \( X_{F,1} \) similarly lowers the price in the foreign market, which decreases the dumping margin \( P_{F,1} - P_{H,1} \). It decreases the expected anti-dumping duty (state \( h_2 \)) in the next period, and induces in equilibrium less aggressive (passive) behavior by the home firm in the home market, which will increase the profit of the foreign firm in the home market.\(^5\) This is a positive strategic effect, which corresponds to the second term of (4). We can make a similar interpretation for (5).

First, an increase in the foreign firm’s current export \( X_{f,1} \) lowers the price in the home market, which increases the dumping margin \( P_{F,1} - P_{H,1} \). It increases both the probability that anti-dumping duties will be in effect in the next period and the magnitude of those duties if enforcement is successful. This brings about a negative direct effect through the increasing impact on future expected anti-dumping duty (state \( h_2 \)), corresponding to the first term of (5). Second, an increase in

\(^5\)If we interpret \( h_1 \left( p_{F,1} - p_{H,1} \right) \) as a “marginal cost” of the foreign firm in the home market, given the (previous period) dumping margin \( p_{F,1} - p_{H,1} \), then it will become easier to understand direct and strategic effects. For such concepts and technique, see Tirole’s IO book (1988).
similarly lowers the price in the home market, which increases the dumping margin $P_{F, 1} - P_{H, 1}$.

It increases the expected anti-dumping duty (state $h_{2}$) in the next period, and induces in equilibrium aggressive behavior by the home firm in the home market, which will reduce the profit of the foreign firm in the home market. This is a negative strategic effect, corresponding to the second term of (5).

2.5 Infinite Horizon Formulation

We formulate the infinite horizon dynamic game, and try to look at a pure strategy Markov Perfect Equilibrium in this dynamic game, taking into consideration the effect of anti-dumping laws.

First, the home firm's and the foreign firm's discounted expected profits at period $t$ are

$$E \left[ \psi_{H,t} \right] = P_{H,t} \left( Q_{H,t} \right) X_{H,t} + \delta \left( P_{H,t+1} \left( Q_{H,t+1} \right) X_{H,t+1} \right) + \delta^2 \left( P_{H,t+2} \left( Q_{H,t+2} \right) X_{H,t+2} \right) + \cdots$$

$$E \left[ \psi_{F,t} \right] = P_{F,t} \left( X_{F,t} \right) X_{F,t} + P_{H,t} \left( Q_{H,t} \right) X_{H,t} - h_{t} X_{F,t} + \delta \left[ P_{F,t+1} \left( X_{F,t+1} \right) X_{F,t+1} + P_{H,t+1} \left( Q_{H,t+1} \right) X_{H,t+1} - h_{t+1} X_{F,t+1} \right] + \delta^2 \left[ P_{F,t+2} \left( X_{F,t+2} \right) X_{F,t+2} + P_{H,t+2} \left( Q_{H,t+2} \right) X_{H,t+2} - h_{t+2} X_{F,t+2} \right] + \cdots$$

$P_{i,j}$ is inverse demand, $i = F$ for the foreign market, $H$ for the home market, and $j$ is the period of time. $X_{i,j}$ is output where $i = F, f, H$ and $j$ indexes time (= 1, 2…). $Q_{H,t} = X_{f,t} + X_{H,t}$ is total output sold at home in period $t$.

The argument of any value function is the state variable, i.e., the expected antidumping duty $h$. Hence, the home firm’s value function can be written as

$$V_{H} \left( h \right) = \max_{x_{H}} \left[ P_{H} \left( X_{H} + X_{f} \left( h \right) \right) x_{H} + \delta V_{H} \left( H \left( P_{F} \left( X_{F} \left( h \right) \right) - P_{H} \left( X_{H} + X_{f} \left( h \right) \right) \right) \right) \right],$$

where we omit time subscripts since the problem is time invariant and the value function $V_{H} \left( h \right)$ should be the same across time so that it should be written without a time subscript, and denote the expected anti-dumping duty by the function $H$ of the price difference $p$, i.e.

$$H \left( p \right) = f \left( p \right) g \left( p \right).$$

A key of this specification is that the foreign firm’s output levels are written as a function of $h$.

The first order condition for the maximization problem is given by
\[ P_H'(X_H + X_f(h))X_H + P_H'(X_H + X_f(h)) - \delta V'_H \left( H \left( P_F(X_F(h)) - P_H(X_H + X_f(h)) \right) \right) \]
\[ \times H'(P_F(X_F(h)) - P_H(X_H + X_f(h))) \times P_H'(X_H + X_f(h)) = 0, \]

which gives us the function \( X_H(h) \). Then, it follows from the envelope theorem that

\[ V'_H(h) = P_H'(X_H(h) + X_f(h))X_f(h) + \delta V'_H \left( H \left( P_F(X_F(h)) - P_H(X_H + X_f(h)) \right) \right) \]
\[ \times H'(P_F(X_F(h)) - P_H(X_H + X_f(h))) \times \left[ P_H'(X_H(h))X_f'(h) - P_H'(X_H(h) + X_f(h))X_f'(h) \right] \]

Note that the second term appears in the right hand side since the current value of the state variable directly affects the valuation from the next period through the foreign firm’s output levels.

Now, suppose hypothetically that the current value of the state variables did not directly affect the valuation from the next period so that the second term would disappear, though dynamic programming analysis should also capture strategic interactions in the periods subsequent to the next one. That is, we have \( V'_H(h) = P_H'(X_H(h) + X_f(h))X_f(h)X_H(h) \), which only captures the effects of current output decisions on strategic interactions in the next period. Then, letting \( H' \) denote the level of the state variable in the next period, we have from the above equations

\[ P_H'(X_H(h) + X_f(h))X_H(h) + P_H'(X_H(h) + X_f(h)) = \delta P_H'(X_H(h') + X_f(h'))X_f'(h')X_H(h') \]
\[ \times H'(P_F(X_F(h)) - P_H(X_H(h) + X_f(h))) \times P_H'(X_H(h) + X_f(h)), \]

which is nothing but equation (1) of the model.

The Foreign firm’s value function can be written as

\[ V_F(h) = \max_{x_F,x_f} \left[ P_F(X_F)X_F + P_H(X_f + X_H(h))X_f - hX_f + \delta V_F \left( H \left( P_F(X_F) - P_H(X_H(h) + X_f) \right) \right) \right] \]

where we omit time subscripts since the problem is time invariant, and the value function \( V_F(h) \) should be the same across time so that it should be written without a time subscript, and denote the expected anti-dumping duty by the function \( H \) of the price difference \( p \). A key of this specification is that the home firm’s output level is written as a function of \( h \).

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The first order conditions of $X_F$ and $X_f$ for the maximization problem are given by

\[
P_F'(X_F)X_F + P_H(X_H(h) + X_f) + \delta V_F' \left( H \left( P_F(X_F) - P_H(X_H(h) + X_f) \right) \right) \times H' \left( P_F(X_F) - P_H(X_H(h) + X_f) \right) \times P_F'(X_F) = 0,
\]

and

\[
P_H' \left( X_H(h) + X_f \right)X_f + P_H(X_H(h) + X_f) - h + \delta V_F' \left( H \left( P_F(X_F) - P_H(X_H(h) + X_f) \right) \right) \times H' \left( P_F(X_F) - P_H(X_H(h) + X_f) \right) \times \left[ -P_H'(X_H(h) + X_f) \right] = 0,
\]

which give us the functions $X_F(h)$ and $X_f(h)$. Then, it follows from the envelope theorem that

\[
V_F'(h) = \left[ P_H'(X_H(h) + X_f(h))X_H'(h) - 1 \right] X_f(h) + \delta V_F' \left( H \left( P_F(X_F(h)) - P_H(X_H(h) + X_f(h)) \right) \right) \times H' \left( P_F(X_F(h)) - P_H(X_H(h) + X_f(h)) \right) \times \left[ -P_H'(X_H(h) + X_f(h)) \right] X_H'(h)
\]

Note that the second term appears in the right hand side since the current value of the state variable directly affects the valuation from the next period through the home firm’s output levels as shown in the above equation.

Now, suppose hypothetically that the current value of the state variables did not directly affect the valuation from the next period so that the second term would disappear. That is, we have $V_F'(h) = \left[ P_H'(X_H(h) + X_f(h)) \right] X_H'(h) - 1 \right] X_f(h)$, which only captures the effects of current output decisions on strategic interactions in the next period, in other words, the so called strategic effect and direct effect. Then, letting $h'$ denote the level of the state variable in the next period, we have from the above equations

\[
P_F'(X_H(h) + X_f(h))X_F(h) + P_H(X_F(h) + X_f(h)) - \delta \left[ P_H'(X_H(h') + X_f(h')) \right]X_H'(h') - 1 \right] X_f(h') \times H' \left( P_F(X_F(h)) - P_H(X_H(h) + X_f(h)) \right) \times P_F'(X_F(h))
\]

which is nothing but equations (2) and (4) of the Two Period model and
which is nothing but equations (3) and (5) of the Two Period model.

2.6 Equilibrium Incentives

2.6.1 Comparison of Equilibrium Incentives in Two Period Framework

The most important factor that distinguishes Markov Perfect (i.e., Closed Loop) equilibria from Open-Loop equilibria (where each firm specifies its output choices (path) over two periods, and chooses Nash strategies) is whether we take into account the strategic effect in deriving the first order condition. In Open-Loop equilibria, the strategic effects do not exist. Therefore, the first order conditions, characterizing the Open-Loop Equilibrium of the model, are in a form that parallels (1) – (5),

\[
\frac{\partial \delta V_{H,1}}{\partial X_{H,1}} = P_{H,1}^\prime X_{H,1} + P_{H,1} = 0 \quad \text{(No strategic effect)} \quad (1)'
\]

\[
\frac{\partial V_{F,1}}{\partial X_{F,1}} = P_{F,1}^\prime X_{F,1} + P_{F,1} + \delta \left( \frac{\partial V_{F,2}}{\partial X_{F,1}} \right) = 0 \quad (2)'
\]

\[
\frac{\partial V_{F,1}}{\partial X_{F,1}} = P_{F,1}^\prime X_{F,1} + P_{F,1} - h_1 + \delta \left( \frac{\partial V_{F,2}}{\partial X_{F,1}} \right) = 0 \quad (3)'
\]

where \( \frac{\partial V_{F,2}}{\partial X_{F,1}} = -P_{F,1}^\prime h_2^1 X_{F,2} \) (Direct effect only) \( (4)' \)

\[
\frac{\partial V_{F,2}}{\partial X_{F,1}} = P_{F,1}^\prime h_2^2 X_{F,2} \quad \text{(Direct effect only)} \quad (5)'
\]

From these first order conditions, we obtain the output levels \( X_{F,1}^{OL}, X_{F,1}^{OL}, X_{H,1}^{OL} \) in the Open-Loop Equilibrium. On the other hand, the static duopoly regime corresponds to \( \delta = 0 \), and so even direct effects disappear in (2)' and (3)'. Therefore, we obtain the following results. (The implications of the strategic effect have already been explained in section 2.3.)
**Proposition:** As for the first period equilibrium outputs under each regime in the Two Period framework, that is $(X^*_F, X^*_{f,1}, X^*_{H,1})$ for Markov Perfect (Closed-Loop) Equilibrium, $(X^*_{OL,F,1}, X^*_{OL,f,1}, X^*_{OL,H,1})$ for Open-Loop Equilibrium, and $(X^*_{SN,F,1}, X^*_{SN,f,1}, X^*_{SN,H,1})$ for Static Nash Equilibrium, the following relation holds:

\[ X^*_{SN,F,1} < X^*_{OL,F,1} < X^*_{SN,f,1} < X^*_{OL,f,1} < X^*_{SN,H,1} < X^*_{OL,H,1} < X^*_{SN,H,1} \]  \( (6) \)

**Proof:** The first and second relations are obviously judged from the signs of the strategic effect and the direct effect in the first order conditions of the three regimes. In the third relation, under both Open-Loop and Static Nash equilibria, the first order conditions for the home firm are the same

\[ \frac{\partial V_{H,1}}{\partial X_{H,1}} = P'_{H,1} X_{H,1} + P_{H,1} = 0 . \]

The order in equilibrium levels of $X_{H,1}$ is determined by the rival’s output levels. Since the outputs $X^*_{f,1}, X^*_{H,1}$ are strategic substitutes in both regimes, and from the fact that $X^*_{SN,f,1} > X^*_{OL,f,1}$, the relation in the proposition holds. Q.E.D.

### 2.6.2 Comparison of Equilibrium Incentives in an Infinite Horizon Framework

As was shown by analysis in section 2.4, a Two Period Model can capture the dynamic effects, consisting of direct and strategic effects, in a simple way. In an Open-Loop equilibrium, only direct effect is taken into consideration when the players make decisions, while in a Markov Perfect Equilibrium, both direct and strategic effects are taken into consideration. They lead to (6). Then, in our framework, the extension from Two Period to Infinite Horizon will strengthen the dynamic effects monotonically. Therefore, we have the following conjecture on equilibrium incentives in an Infinite Horizon Framework.

**Conjecture:** As for equilibrium outputs under each regime in an Infinite Horizon framework, we have

\[ X^{SN}_F < X^{OL}_F < X^{*}_F, X^{SN}_f > X^{OL}_f > X^{*}_f, X^{SN}_{H} < X^{OL}_{H} < X^{*}_{H} \]  \( (6)' \)
and the Markov Perfect (closed loop) equilibrium levels \( \left( X_F^*, X_H^* \right) \) are greater in Infinite Horizon than in Two Period, while \( X_f^* \) is smaller in Infinite Horizon than in Two Period. As for the open loop equilibrium levels, we have the similar result such that \( \left( X_F^{OL}, X_H^{OL} \right) \) are greater in Infinite Horizon than in Two Period, while \( X_f^{OL} \) is smaller in Infinite Horizon than in Two Period.

3. Conclusion

The aim of this paper was to examine quantity-setting behavior in the presence of anti-dumping law in an infinite horizon international duopoly model. Firms’ quantity setting for the current period affects the expected anti-dumping duty levied on imports in the next period. Therefore, firms decide their individual output levels taking into account their impact on strategic interactions from the next period onwards. Considering hypothetically the impact of firms’ current output decisions only on their strategic positions in the next period, in other words, ignoring effectively those in the periods subsequent to the next period, we gained an understanding of the relationship between two period and infinite horizon formulations. Then, we compared firms’ output decisions across the closed-loop (Markov Perfect) equilibrium, open-loop equilibrium, and the static Nash equilibrium, in order to analyze the dynamic links of output decisions caused by anti-dumping law, and presented a conjecture on the comparison of equilibrium incentives in an Infinite Horizon framework.
REFERENCES


*European Economic Review*. 31, 947-68.


